# Complete spectroscopy in high-spin cranking calculations 

I. Ragnarsson ${ }^{1, \mathrm{a}}$, C. Andreoiu ${ }^{2, \mathrm{~b}}$, T. Døssing ${ }^{3}$, C. Fahlander ${ }^{2}$, D. Rudolph ${ }^{2}$, and S. Åberg ${ }^{1}$<br>${ }^{1}$ Division of Mathematical Physics, Lund Institute of Technology, P.O. Box 118, SE-221 00 Lund, Sweden<br>${ }^{2}$ Department of Physics, Lund University, P.O. Box 118, SE-221 00 Lund, Sweden<br>${ }^{3}$ The Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17, DK-2100 Copenhagen, Denmark

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#### Abstract

Exact and approximate quantum numbers of the cranked Nilsson-Strutinsky (CNS) formalism are exploited to calculate excited bands in fixed configurations with the energy of the individual bands minimized with respect to deformation for all spin values. The formalism is applied to ${ }^{59} \mathrm{Cu}$, where all bands which appear important in the decay out of the superdeformed band are calculated.


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A large number of rotational bands and high-spin states have recently been identified in the nucleus ${ }^{59} \mathrm{Cu}[1]$. In order to describe most of these states, it appears necessary to consider a space including at least the full $\mathcal{N}=3$ shell and the $g_{9 / 2}$ subshell. This space is too big for standard shell model calculations. Thus, it appears more suitable to tackle this nucleus with a mean-field approach. Using such an approach, it is necessary to be able to fix configurations so that the rotational bands can be followed over large spin ranges. This is especially important in this light-mass region where most bands appear to undergo large shape changes as functions of spin. Indeed, many bands are observed to terminate [2] in a non-collective state with a large jump in deformation between the terminating $I_{\max }$ state and the $\left(I_{\max }-2\right)$ state.

In the cranked Nilsson-Strutinsky (CNS) approach of refs. [3,4], the orbitals are labeled not only by the exact quantum numbers parity and signature, but also by approximate quantum numbers. Thus the main oscillator quantum number $\mathcal{N} \equiv \mathcal{N}_{\text {rot }}$, corresponding to the total number of quanta in the rotating harmonic-oscillator basis, is treated as pure, and in each $\mathcal{N}$-shell, a distinction is made between the high- $j$ intruder orbitals and the other (low- $j$ ) orbitals. Using these "quantum numbers", the configurations of ${ }^{59} \mathrm{Cu}$ are labeled by the number of (high- $j$ ) $f_{7 / 2}$ holes and $g_{9 / 2}$ particles for protons and neutrons, i.e. $\left[p_{1} p_{2}, n_{1} n_{2}\right]=\pi\left(f_{7 / 2}\right)^{-p_{1}}\left(g_{9 / 2}\right)^{p_{2}} \nu\left(f_{7 / 2}\right)^{-n_{1}}\left(g_{9 / 2}\right)^{n_{2}}$. Note that the number of low- $j \mathcal{N}=3$ particles will then be fixed by the condition of constant proton and neutron number, $Z=29$ and $N=30$. For $Z \approx N$ nuclei, it is

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Fig. 1. Illustration of possible configurations in ${ }^{59} \mathrm{Cu}$ showing the holes in the highest $f_{7 / 2}$ orbitals, the particles in the two lowest orbitals of ( $f_{5 / 2}, p_{3 / 2}$ ) character labeled ( $f p$ ), and the particles in $g_{9 / 2}$. Plus and minus signs are used for signature $\alpha=+1 / 2$ and $\alpha=-1 / 2$ states, respectively.
often difficult to distinguish between proton and neutron excitations. Therefore, it is sometimes more appropriate to specify only the total number of holes $q_{1}$ and particles $q_{2}$. In this case we refer to the $\left[p_{1} p_{2}, n_{1} n_{2}\right]$ configuration as $\left[q_{1} \backslash q_{2}\right.$ ] where $q_{1}=p_{1}+n_{1}$ and $q_{2}=p_{2}+n_{2}$.

Possible occupations of the orbitals in ${ }^{59} \mathrm{Cu}$ are illustrated in fig. 1. One can note that the shorthand notation introduced above does not fully classify a configuration. It is necessary to specify also the signature of the different particles. All configurations in fig. 1 are of the type $[00,11]$. The one to the left is low in energy with all three $\left(f_{5 / 2}, p_{3 / 2}, p_{1 / 2}\right) \equiv(f p)$ particles in the lowest proton and neutron orbitals of this kind. In the middle panel, the ( $f p$ ) proton is excited to a higher orbital. In the right panel, the two ( $f p$ ) neutrons have the same signature, so that one of them must be placed in the second lowest $(f p)$ orbital. Note that the filling of orbitals in the left and middle panels corresponds to the yrast band and an excited band of the same configuration.
Observed configurations in ${ }^{59} \mathrm{Cu}\left({ }^{56} \mathrm{Ni}\right.$ core)

|  | $\left(\mathrm{g}_{9 / 2}\right)^{0}$ | $\left(\mathrm{~g}_{9 / 2}\right)^{1}$ | $\left(\mathrm{~g}_{9 / 2}\right)^{2}$ | $\left(\mathrm{~g}_{9 / 2}\right)^{3}$ |
| :---: | :---: | :---: | :---: | :---: |
| closed core | $1 \mathrm{a}, 1 \mathrm{~b}$ | 2 |  |  |
| $\left(\mathrm{f}_{7 / 2}\right)^{-1}$ | 1 c | 3 |  |  |
| $\left(\mathrm{f}_{7 / 2}\right)^{-2}$ | 1 d | 4 | 7 |  |
| $\left(\mathrm{f}_{7 / 2}\right)^{-3}$ |  |  | 8 |  |
| $\left(\mathrm{f}_{7 / 2}\right)^{-4}$ |  |  | 6 | $5, \mathrm{SD}$ |

Fig. 2. Configurations assigned to the observed "rotational" bands in ${ }^{59} \mathrm{Cu}$ labeled $1 \mathrm{a}, 1 \mathrm{~b}, \ldots, 8$ in ref. [1].

The configurations of ${ }^{59} \mathrm{Cu}$ can be classified as in fig. 2. The rows specify the number of holes in $f_{7 / 2}$ orbitals and the columns the number of $g_{9 / 2}$ particles. The numbers in the panels of fig. 2 indicate that these configurations have been assigned to the different observed bands numbered as $1 \mathrm{a}, 1 \mathrm{~b}, 1 \mathrm{c}, 1 \mathrm{~d}, 2$, etc. in ref. [1].

With our detailed configuration assignment and with the possibility to consider excited bands in fixed configurations, it becomes possible to calculate all states in some energy range above yrast. This could be important when considering, for example, the decay out of superdeformed (SD) bands. If this decay is treated in detail, it is necessary to know all states having energies in the same range or below the SD band. We will demonstrate this by considering the decay out of the SD band in ${ }^{59} \mathrm{Cu}$, which has been mapped out in great detail in experiment [1]. The proton decay is discussed by D. Rudolph in a separate contribution [5]. Of interest for us is the $\gamma$ emission, where around $90 \%$ of the full intensity has been observed. Mainly positive-parity states, including the lower-spin states of bands 3 and 4, are involved in the decay which appears to present an intermediate case between the highly fragmented statistical decay of SD bands in heavy nuclei and the more direct decay in many lighter nuclei.

The superdeformed band is assigned as $[4 \backslash 3]$ (cf. fig. 2). All positive-parity bands which are calculated lower in energy have only one $g_{9 / 2}$ particle. The SD band is a 2 p 2 h excitation relative to the observed $[2 \backslash 1]$ configuration (band 4), and also with respect to the unobserved [3\1] and $[4 \backslash 1]$ configurations, but it is a $3 \mathrm{p}-3 \mathrm{~h}$ excitation with respect to the $[1 \backslash 1]$ configuration (band 3 ). This suggests that the SD $[4 \backslash 3]$ band can only couple directly with configurations of the type [ $4 \backslash 1$ ], $[3 \backslash 1]$ and $[2 \backslash 1]$.

Calculated energies of all bands which come below or close to the SD band for $I \leq 25 / 2$ are shown in fig. 3(a). There are three $[2 \backslash 1]$ configurations of this kind (three because one of the curves includes two degenerate bands). All other [ $2 \backslash 1]$ configurations are calculated at a higher energy. In a similar way, we only need to consider three [3\1] configurations while all $[4 \backslash 1]$ configurations are calculated at a higher energy than the SD band. There are many $[1 \backslash 1]$ configurations at low energy; six low-lying drawn by full lines, four bands corresponding to proton excitations (cf. middle panel of fig. 1), and five neutron excited bands. Finally, there are seven other "higher bands" formed as yrast in configurations with the particles distributed unevenly over the two signatures (cf. right panel of fig. 1).


Fig. 3. (a) Calculated energies of rotational bands in ${ }^{59} \mathrm{Cu}$ with a rigid-rotation reference subtracted and with the terminating states encircled. The relative energies of configurations assigned to the SD band and the observed bands 3 and 4, i.e. the lowest [ $4 \backslash 3$ ], $[1 \backslash 1]$ and $[2 \backslash 1]$ configurations, have been fitted to experimental energies. (b) The lowest [4\3], [3\1], and [2\1] bands are shown along a "minimum energy path" in the $(\varepsilon, \gamma)$ plane for $I=12.5$ and 16.5 . Assuming a 0.5 MeV zero-point energy in the SD well, the decay barriers are indicated in grey.

According to fig. 3(a), the SD band is very close in energy to states to which it may couple by $2 \mathrm{p}-2 \mathrm{~h}$ matrix elements, first at $I=33 / 2$, where it comes close to the low-lying [ $2 \backslash 1$ ] bands, and then again to [ $3 \backslash 1$ ] bands at $I=25 / 2$. At both spins, the coupling also involves a change in deformation, which is illustrated in fig. 3(b). Here the energy curves for the relevant configurations are drawn along a straight line in the $(\varepsilon, \gamma)$-plane. In this figure, the SD band appears to be shielded from the $[2 \backslash 1]$ states at $I=33 / 2$ by a barrier, but the equivalent barrier separating it from the $[3 \backslash 1]$ states at $I=25 / 2$ is much weaker. This may explain the experimental finding that the SD band remains pure and the intensity stays in the band at $I=33 / 2$, whereas it fragments at $I=25 / 2$. According to this picture, the $[3 \backslash 1]$ states act as doorway states for the SD decay-out. In the next stage, the $[3 \backslash 1]$ bands will in turn mix with the $[1 \backslash 1]$ and $[2 \backslash 1]$ bands, suggesting a fragmentation of the decay in general agreement with experiment, see refs. $[6,7]$.

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[^0]:    ${ }^{\text {a }}$ e-mail: ingemar.ragnarsson@matfys.lth.se
    b Present address: Department of Physics, University of Guelph, Guelph, Ontario, Canada N1G 2W1.

